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Better synchronizability in generalized adaptive networks

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In this paper, to study the interaction between network structure and dynamical property in the context of synchronization, a previously proposed adaptive coupling method is generalized where the coupling strength of a node from its neighbors not only develops adaptively according to the local synchronization property between the node and its neighbors (dynamical part) but also is modulated by its local structure, degree of the node with the form $1/k_i^{\alpha}$ (topological part). We can show both numerically and analytically that the input coupling strength of the network after adaptation displays a power-law dependence on the degree, $k^{-\theta}$, where the exponent θ is controlled by α as $\theta=(1+\alpha)/2$. Compared to the original adaptive coupling method, after the addition of modulation, the distribution of the node's intensity is tunable and can be more homogenous with $\alpha\approx 1$, which results in better synchronizability. It is also found that the synchronization time can shrink greatly. Our theoretical work in the context of synchronization provides not only a deeper understanding of the interplay between structure and dynamics in real world systems, such as opinion formation and concensus, but also potential approaches to manipulate the global collective dynamics through local adaptive control.

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Since the discovery and modeling of small world and scale-free networks ten years ago [1,2], the dynamics in complex networks have been a research topic of great attention. These dynamics include the epidemic and percolation process, cascading behavior, traffic, opinion formation, synchronization and consensuses, etc. [3–7]. Now it is well known that not only the topology of the networks but also the weights of nodes and strengths of links have great effects on the dynamics in the networks [8–11]. Most of these studies mainly considered the effects of structure on the dynamics, while the dynamics have no effects on the network structure, i.e., no matter how the dynamics change, the topology and the weights and strengths will keep unchanged. However, in most realistic dynamical networks, the dynamics can sometimes induce changes in network structure. For example, in airport networks of airports and airlines, the airlines affect the transportation and the demand of transportation between different airports may result in the increasing or decreasing of the strengths of airlines or even the emergence of new airlines. Another typical example is in the social network where an epidemic spreads among the people. It is obvious that the network structure will affect the direction and speed of the epidemic spreading and meanwhile the susceptible will avoid contact with the infected by rewiring their network connections, thus the network structure is changed [12]. Similar interplay between structure and dynamics happens in some other realistic systems where synchronization is relevant. For example, in neural systems, the synapses are strengthened when two neural populations synchronize their activity, which is known as the Hebbian learning rule and is the mechanism underlying learning and memory of the brain. It has been shown that applying such Hebbian-like rule (coupling strength increases faster when synchronization is stronger) to large network of oscillators will lead to the formation of dynamical and structural clusters [13]. In other systems, global synchronization of the whole network is more desirable, for example, to achieve consensus on some opinion. Base on our intuition and observation, we know that the strategy often taken in such systems is "following the majority." The agents try to approach those individuals who have different opinions from their own and persuade them to follow their common opinion, while they do not have to put much effort to enhance the interaction with the others already sharing similar opinions.

The impact of this type of adaptation on network structure has been investigated recently [14–16] in the context of synchronization of complex networks. Mathematically, we can represent this type of adaptation by a rule opposite to the Hebbian one: the coupling strength increases proportional to the synchronization difference in order to suppress the difference. In particular, we previously proposed an adaptive coupling method where the input coupling strength of a node from its neighbors develops adaptively according to the local synchronization property between the node and its neighbors [14]. In such adaptive networks, when complete synchronization is achieved, the coupling strength becomes stable and weighted, which obeys a power-law form with respect to the node degree. This makes the network synchronizability better than the symmetrical coupling method [17,18] but not yet in the optimal situation. In our previous studies on the synchronization in weighted random networks, we had proved that for random networks with a large minimum degree, larger average degree and more homogenous distribution of node's intensity will ensure better network synchronizability, where the intensity of a node is defined as its total input coupling strengths [8,19]. For a given average degree, the network synchronizability will be optimized when all the nodes' intensities are equal. In the adaptive networks we pro-

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posed, it was found that the exponent in the power-law distribution of the strength is not sensitive to the dynamical model, network size, average degree, the degree distribution of the network and other factors, which results in the heterogenous distribution of the node intensity, as a result the network synchronizability is not yet the best. Thus, to find out what could further influence the exponent and make the distribution of node intensity homogeneous is of great importance for the understanding the interplay between structure and dynamics and for the manipulation of the synchronizability of adaptive networks. The adaptive coupling methods mentioned above are only based on the information of dynamical states and overlook the effects of network structure, which is an critical property of complex networks and has been proved to play an important role in the manipulation of network synchronizability [8,20–22]. Previous analysis has shown that the weight of the links in realistic networks are correlated, so that the capacity or intensity of the node, denoted by the total weight, scales with the degrees. Thus, in this paper, we consider the interaction between network structure and dynamical state in dynamical network, and investigate how the network synchronizability will change. We hope the distribution of the coupling strengths will be affected if the coupling method takes both the information of dynamical state and network structure into account.

In this paper, we generalize our former adaptive coupling method introduced in [14] to incorporate the modulation of coupling strength by the degree of each node with the form $1/k_i^\alpha$. This is again motivated from our intuitive experience in social dynamics that the capacity of any agent may not increase as fast as degree. Nodes with larger number of neighbors may have to put less effort to each of the neighbor due to limitation of its capacity. How this limitation in the capacity influences the collective dynamics and structure in adaptive network is an interesting question. We are able to show analytically that the input strength of the network after adaptation displays a power law dependence on the degree, $k_i^{-\theta}$ and the exponent θ is controlled by α as $\theta=(1+\alpha)/2$. The analysis is confirmed by numerical simulations. As a result, the distribution of node's intensity becomes $k^{1-\theta}$, and can be regulated as homogenous as possible, which will lead to enhanced network synchronizability. Moreover, we find that with the enhancement of synchronization stability, the shortest synchronization time will also shrink, and the networks are in the most synchronizable state.

The dynamical equation of the ith node in an N coupled identical chaotic oscillator network is as following:

$$\dot{\mathbf{x}}_i = \mathbf{F}(\mathbf{x}_i) + \sum_{j=1}^{N} a_{ij} W_{ij}[\mathbf{H}(\mathbf{x}_j) - \mathbf{H}(\mathbf{x}_i)] = \mathbf{F}(\mathbf{x}_i) - \sum_{j=1}^{N} G_{ij}\mathbf{H}(\mathbf{x}_j), \tag{1}$$

where $\mathbf{x}_i$ is the state of node i, $i=1,\ldots,N$, $\mathbf{F}(\mathbf{x})$ is the dynamics of individual oscillator, $\mathbf{H}(\mathbf{x})$ is the linear output function which decides how a node is coupled by its neighbors, $A=a_{ij}$ is the adjacent matrix, and $W_{ij}>0$ is the coupling strength from node j to node i if they are connected. $G=(G_{ij})$ is the coupling matrix and is defined as

$$G_{ij} = \begin{cases} -a_{ij}W_{ij} & \text{for } i \neq j \\ \sum_{j=1}^{N} a_{ij}W_{ij} & \text{for } i = j. \end{cases} \tag{2}$$

In most of the previous studies, the coupling strengths will not be affected by the dynamics, i.e., the network is unchanged [17–23], thus, the elements of the coupling matrix can be written as $G_{ij}=\sigma G_{ij}^0$, where G_{ij}^0 is fixed and σ is the overall coupling strength. In this paper, we focus on the case that the coupling strength W_{ij} will change adaptively with the dynamical property of the network.

Before presenting the generalized adaptive networks (GANs), we give an introduction of the original adaptive networks (ANs). In ANs, the input coupling strength W_{ij} is controlled by the local synchronization properties of the nodes. In particular, we suppose that the strength to a node i from all its k_i neighbors increases uniformly among the k_i connections, in order to suppress its difference Δ_i from the mean activity of its neighbors, namely,

$$W_{ij}(t) = V_i(t), \quad \dot{V}_i = \gamma\Delta_i/(1+\Delta_i), \tag{3}$$

where $\Delta_i=|\mathbf{H}(\mathbf{x}_i)-(1/k_i)\Sigma_j a_{ij}\mathbf{H}(\mathbf{x}_j)|$ is the difference of the output states between the node i and the average activity of its neighbors and $\gamma>0$ is the adaptation parameter. In this coupling method, the input coupling strength of each node depends only on the local dynamical property, here, we will modify it up by introducing the effect of network structure into the adaptive networks. In all the weighted coupling methods that make use of the network structure information, the method introduced in our previous work [8] is probably the simplest but effective one, where the input coupling strength of each node is defined as $1/k_i^\alpha$, and $\alpha=1$ corresponds to the normalized Laplacian matrix, at which point the network synchronizability is optimized. Thus, we design the generalized adaptive coupling method as

$$\dot{V}_i' = \dot{V}_i/k_i^\alpha = \gamma'\Delta_i/(1+\Delta_i), \tag{4}$$

where α is tunable and $\gamma'=\gamma/k_i^\alpha$ is the generalized adaptation parameter. When $\alpha>0$ nodes with larger degrees will be coupled more slightly, while $\alpha<0$ corresponds to the reverse case and when $\alpha=0$, the coupling method degenerates into the original one and $V'=V$.

For (generalized) adaptive networks, when the networks reach the synchronization state, the coupling strengths will be stable, thus, the dynamical equation can be written as

$$\dot{\mathbf{x}}_i = \mathbf{F}(\mathbf{x}_i) - \sigma\sum_{j=1}^{N} G_{ij}^0\mathbf{H}(\mathbf{x}_j). \tag{5}$$

According to the master stability function [17,18], the stability of this synchronization state can be measured by the eigenvalues of the coupling matrix G^0. Although the coupling matrix is asymmetric, similar to [8] we can prove that all the eigenvalues are none-negative real values, and more specifically, when the network is connected, there will be only one zero value, thus all the eigenvalues can be ranked as $0=\lambda_1<\lambda_2\leq\lambda_3\leq\cdots\leq\lambda_N$. In (generalized) adaptive networks, when

$$\alpha_1 < \sigma\lambda_2 \leq \cdots \leq \sigma\lambda_N < \alpha_2, \tag{6}$$

the synchronization state will be stable, where α_1 and α_2 is determined by **F** and **H**, which have nothing to do with the network structure. Thus for a network, larger λ_2 and smaller λ_N means better synchronizability. In some cases, α_2 is infinity and the synchronization region is half bounded, and the network synchronizability is enhanced by increasing the value of λ_2. In the other cases, α_2 takes finite value and the synchronization region is bounded. When the eigenratio $R=\lambda_N/\lambda_2<\alpha_2/\alpha_1=\beta$, the network can be synchronized and smaller R indicates better network synchronizability.

In the (generalized) adaptive networks, assume that the initial coupling strength along each link is a random small value and the chaotic oscillators iterate from random initial states. Then, Δ_i in Eq. (3) is about uniform for all the nodes in the beginning and the input coupling strength of each node will increase uniformly in the whole ANs in a short period, i.e., $W_{ij}=V_i(t)\approx\gamma t$. But in GANs, from the very beginning the input coupling strengths will be very different because of the differences between node degrees, i.e., $W_{ij}=V_i'(t)\approx\gamma' t=(\gamma/k_i^{\alpha})t$, and by comparison, the input coupling strengths for nodes with smaller degrees will increase faster in case of $\alpha>0$. If all the parameters are taken properly, the networks can reach complete synchronization state and the input coupling strength for each node will stabilize at different constants $G_{ij}^0=V_i'^0$.

To give a clear picture of the synchronization process of the whole network, we let the nodes iterate in the well-known Barabási-Albert (BA) scale-free networks [2]. We will investigate how the input coupling strengths of different nodes change from about zero to constants, what affects the synchronization time, and how the synchronization stability will be after the network reaches complete synchronization state. In the simulation, the node in network is represented by the chaotic Rössler oscillator, $\mathbf{x}=(x,y,z)$ and $\mathbf{F}(\mathbf{x})=[-y-z,x+0.2y,z(x-7.0)+0.2]$. Here, we take the output function $\mathbf{H}=(1,0,0)$, which makes the synchronization region bounded. In Fig. 1, the change of input coupling strength with time for three nodes with large, middle and small degrees are plotted. It is seen that at the very beginning, there is almost no difference in the input coupling strength between nodes with different degrees in AN, but the differences are obvious for the nodes in GANs, which confirms our above analysis. The intensive investigation in [14] found that the input coupling strengths obey power-law form with node degrees after the network reaches complete synchronization, and the power exponent is independent of network size, average degree, adaptation parameter γ, and the degree distribution of the network. However, in Fig. 1 it is obvious that the distribution of input coupling strength in GAN is more heterogeneous than that in AN, which results from the regulation of input coupling strength. Next, we will investigate how the distribution of input coupling strength is affected by the parameter α.

In the simulation, we find that the distribution of stable input coupling strength display power-law relations with the node's degree for any value of α, i.e., $V_i'\sim k^{-\theta(\alpha)}$, but the exponents θ is a function of parameter α. In Fig. 2(a) we

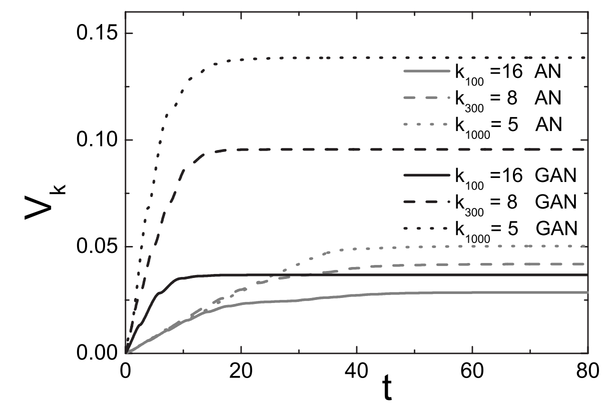

FIG. 1. The variation of input coupling strength as a function of time for three nodes with typical degrees: $k_{100}=16$ (large degree), $k_{300}=8$ (middle degree) and $k_{1000}=5$ (small degree). The nodes are coupled by adaptive coupling method with adaptation parameter $\gamma=0.006$ (gray lines) and generalized adaptive coupling method with $\gamma=0.3$ and $\alpha=1.0$ (black lines), respectively. The network is BA scale-free network with size $N=1000$ and minimum degree $M=5$.

present the change of input coupling strength with respect to the degree k in GAN with $\alpha=0.0$ (AN) and $\alpha=1.0$ at different adaptation parameter γ. It is evident that the exponent θ of GAN at $\alpha=1.0$ is much larger than that at $\alpha=0.0$, which is independent of the adaptation parameter γ. Then we investigate the relation between the exponent θ and parameter α and we find that θ increases almost linearly with α, which means larger α makes the distribution of input coupling strengths more heterogenous [Fig. 2(b)]. This result is obvious because at the very beginning, the input coupling strength for each node increases proportional to its generalized adaptation parameter $\gamma'=\gamma/k_i^{\alpha}$, whose value depends not only on the parameter α but also on the node degree k_i, and with α's increasing, the differences between generalized

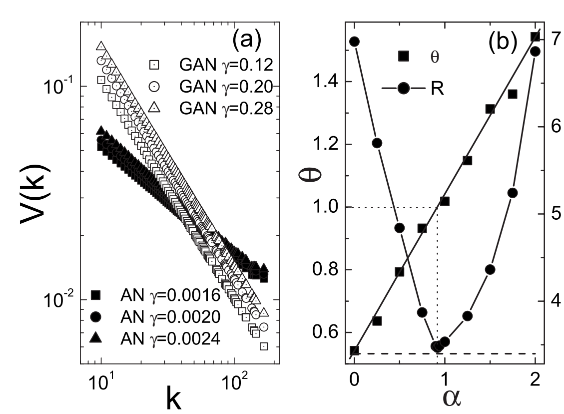

FIG. 2. (a) Relations between the input coupling strength $V(k)$ and degree k in AN and GAN at $\alpha=1.0$. The distribution of V is a power function of k: $V(k)\propto k^{-\theta}$. (b) The change of θ and R with α. The straight line is the fitting line of θ and the dash line shows the eigenratio for the same networks that coupled by normalized Laplacian matrix. The simulation is based on BA scale-free network with 1000 nodes and minimum degree 10. Each node is obtain after the averaging over 50 network configurations and 50 initial states for each configuration.

adaptation parameters of nodes with different degrees become larger and larger, thus, the distribution of stable input coupling strength will become more and more heterogenous, corresponding to increasing θ. We find that the results can be well fitted by

$$\theta = \theta_0 + 0.5\alpha, \tag{7}$$

where θ_0 is the power exponent when $\alpha=0.0$ (ANs), as plotted in Fig. 2(b) (the straight line).

Next, we can show from theoretical analysis that Eq. (7) is satisfied. First let us linearize Eq. (1) around the local mean-field $\bar{\mathbf{x}}_i = 1/k_i \Sigma_j a_{ij}\mathbf{x}_j$. We can take the linear approximation for $\mathbf{H}(\mathbf{x}_j)$ and $\mathbf{H}(\mathbf{x}_i)$ in Eq. (1) as

$$\mathbf{H}(\mathbf{x}_j) \approx D\mathbf{H}(\bar{\mathbf{x}}_i)(\mathbf{x}_j - \bar{\mathbf{x}}_i), \tag{8}$$

$$\mathbf{H}(\mathbf{x}_i) \approx D\mathbf{H}(\bar{\mathbf{x}}_i)(\mathbf{x}_i - \bar{\mathbf{x}}_i). \tag{9}$$

Denoting $\Delta\mathbf{x}_i = \mathbf{x}_i - \bar{\mathbf{x}}_i$, we can see that the network coupling term in Eq. (1) reads

$$\sum_{j=1}^{N} a_{ij} W_{ij}[\mathbf{H}(\mathbf{x}_j) - \mathbf{H}(\mathbf{x}_i)] = V_i(t) \sum_{j=1}^{N} a_{ij}[\mathbf{H}(\mathbf{x}_j) - \mathbf{H}(\mathbf{x}_i)] \approx -k_i V_i(t) D\mathbf{H}(\bar{\mathbf{x}}_i)\Delta\mathbf{x}_i. \tag{10}$$

As a result, we obtain the following linear evolution equation for $\Delta\mathbf{x}_i$,

$$\Delta\dot{\mathbf{x}}_i = [D\mathbf{F}(\bar{\mathbf{x}}_i) - D\mathbf{H}(\bar{\mathbf{x}}_i) k_i V_i(t)]\Delta\mathbf{x}_i. \tag{11}$$

We can go further to assume that the dynamics is already close to the synchronization manifold $\mathbf{s}$, so that the local mean field $\bar{\mathbf{x}}_i \approx \mathbf{s}$, and Eq. (11) can be further approximated as

$$\Delta\dot{\mathbf{x}}_i = [D\mathbf{F}(\mathbf{s}) - D\mathbf{H}(\mathbf{s}) k_i V_i(t)]\Delta\mathbf{x}_i. \tag{12}$$

Note that Eq. (12) only depends on the index i. This means that the above approximations we applied have decoupled the evolution of $\Delta\mathbf{x}$ for each node. Equation (12) has the same form for each node if the index i is dropped, and we can represent this common form as

$$\dot{\xi} = [D\mathbf{F}(\mathbf{s}) - \epsilon D\mathbf{H}(\mathbf{s})]\xi, \tag{13}$$

where $\xi = \Delta\mathbf{x}_i$ and $\epsilon = k_i V_i(t)$. Written in this way, this equation has the same form as the generic mode equation in the framework of master stability function [18]. In this framework, when ϵ is a time-invariant constant, the magnitude $|\xi|$ evolves asymptotically as

$$|\dot{\xi}| = \Lambda(\epsilon)|\xi|, \tag{14}$$

where $\Lambda(\epsilon)$ is the master stability function, i.e., the largest Lyapunov exponent of Eq. (13) with constant ϵ. We note that for many oscillators $\mathbf{F}$ and coupling functions $\mathbf{H}$, $\Lambda(\epsilon)$ first decreases with ϵ almost linearly in a range of ϵ after crossing the threshold α_1 (cf. Fig. 1 of [18]), i.e., $\Lambda(\epsilon) \sim -\epsilon$.

In adaptive network, the coupling strength V_i increases from a small value around zero. In the very beginning, there is no synchronization, and Δ_i is quite uniform for all the nodes; according to Eq. (4), V_i increases with a constant rate proportional to $\gamma' = \gamma / k_i^{\alpha}$ till $k_i V_i(t)$ exceeds the synchronization threshold α_1 where Δ_i starts to decrease due to synchronization. The increase in V_i slows down and quickly approaches to a constant value (see Fig. 1). Therefore, the evolution of ξ in Eq. (13) with time-varying $\epsilon = k_i V_i(t)$ can be approximated by Eq. (14) basing on the master stability function under constant ϵ. Consequently, we obtain $|\Delta\dot{\mathbf{x}}_i| \approx \Lambda(\epsilon)|\Delta\mathbf{x}_i| \sim -k_i V_i(t)|\Delta\mathbf{x}_i|$. For a linear coupling function $\mathbf{H}(\mathbf{x}) = \mathbf{C}\mathbf{x}$, like the one considered here, we also have $\Delta_i = |\mathbf{H}(\mathbf{x}_i) - (1/k_i)\Sigma_j a_{ij}\mathbf{H}(\mathbf{x}_j)| = |\mathbf{C}| \times |\Delta\mathbf{x}_i|$. Thus, we get linearized evolution equations for Δ_i as

$$\dot{\Delta}_i(t) \sim -k_i V_i(t)\Delta_i(t). \tag{15}$$

From Eq. (4), we also get the following linear equation for the evolution of V_i,

$$\dot{V}_i(t) \approx \frac{\gamma}{k_i^{\alpha}}\Delta_i(t). \tag{16}$$

In the following we will solve Eqs. (15) and (16). Let $\dot{\Delta}_i(t) = -\rho k_i V_i(t)\Delta_i(t)$, where $\rho > 0$ is a constant. From Eqs. (15) and (16), we have

$$[V_i^2(t)]' \approx \frac{2\gamma}{k_i^{\alpha}} V_i(t)\Delta_i(t) = -\frac{2\gamma}{\rho k_i^{\alpha+1}}\dot{\Delta}_i(t). \tag{17}$$

Integrating both sides of Eq. (17) from 0 to t, it follows that

$$V_i^2(t) \approx -\frac{2\gamma}{\rho k_i^{\alpha+1}}[\Delta_i(t) - \Delta_i(0)] + V_i^2(0). \tag{18}$$

Taking Eq. (18) into Eq. (16), we get

$$\dot{V}_i(t) \approx \frac{\gamma}{k_i^{\alpha}}\left\{-\frac{\rho k_i^{\alpha+1}[V_i^2(t) - V_i^2(0)]}{2\gamma} + \Delta_i(0)\right\} \approx -\frac{\rho k_i}{2}\left[V_i^2(t) - V_i^2(0) - \frac{2\Delta_i(0)\gamma}{\rho k_i^{\alpha+1}}\right]. \tag{19}$$

By solving Eq. (19), one obtains

$$\lim_{t\to+\infty} V_i(t) \approx \sqrt{V_i^2(0) + \frac{2\Delta_i(0)\gamma}{\rho k_i^{\alpha+1}}} \sim \gamma^{1/2} k_i^{-(\alpha+1)/2}, \tag{20}$$

by taking $V_i(0) \approx 0$ and $\Delta_i(0)$ a similar constant for different nodes (random initial condition), one finally gets

$$V(k) \sim k^{-\theta}, \tag{21}$$

$$\theta(\alpha) = \frac{\alpha+1}{2}, \tag{22}$$

which suits Eq. (7) very well. The analysis also predicts that $\theta_0 = 0.5$.

We also investigate whether the distribution of the initial states of the input coupling strengths will affect their final distribution. In simulation, we have tried some distribution of initial coupling strength $V_i(0) = \epsilon_i$, such as random distribution, Poisson distribution and power-law distribution, the result always remains the same as long as the initial coupling strengths are small enough.

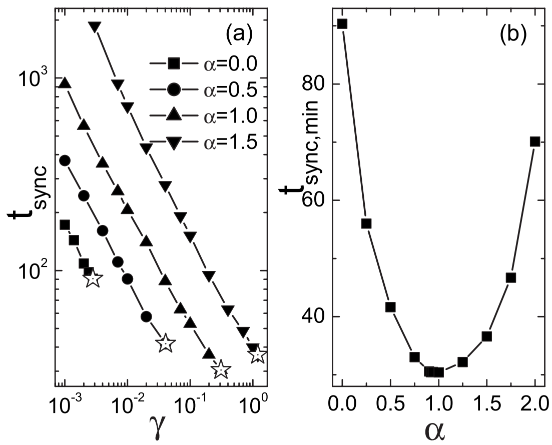

FIG. 3. (a) The synchronization time t_{sync} vs adaptation parameter γ at different α. The stars are γ_c for corresponding parameters. (b) The change of shortest synchronization time with the parameter α. The other network parameters are the same as in Fig. 2.

To investigate the network synchronizability, we obtain the intensity of nodes in stable GANs, $S_i = k_i V_i \sim k_i^{1.0-\theta}$, when $1.0-\theta = 0.0$, i.e., $\alpha = 2\times(1.0-\theta_0)$, all the nodes have the same intensity, and the network synchronizability will be optimized, while larger or smaller α will degrade the network synchronizability. The eigenratio R as a function of α is also shown Fig. 2(b). In our case, $\theta_0 \approx 0.54$, which is very close to the analytical prediction in Eq. (22). The small difference could mainly due to the fact that $\Lambda(\epsilon)$ is not an exact linear function of ϵ (cf. Fig. 1 of [18]). Accordingly, $\alpha \approx 0.92$ is expected to be the optimal point. This is confirmed by the simulation result: R is minimal at the point and is almost the same as that of the networks coupled by normalized Laplacian matrix, where each node is coupled at strength $1/k_i$ [dash line in Fig. 2(b)].

Synchronization time is an important character of synchronization performance of the network, thus, we analyze the change of synchronization time in GANs. In our system two parameters γ and α can change the synchronization time. In the simulations, synchronization is supposed to be achieved when the standard deviation of the states of oscillators is less than 10^{-6} and stay less than 10^{-6} for a period 600. The time to reach this threshold is taken as the synchronization time.

If the synchronization region is bounded, when $\gamma < \gamma_c$ the network will synchronize and the larger the γ is the faster the network reaches complete synchronization state. Then we investigate the effect of parameter α on the synchronization time. In Fig. 3(a), we show the synchronization time t_{sync} with respect to parameter γ at different α. It is clearly seen that for a given α, the synchronization time will decrease with the increasing of γ till γ_c, beyond which the network will not synchronize, and at γ_c^- the synchronization time is the shortest. Just at this point, $\lambda_N \approx \alpha_2$, thus $\lambda_2 \approx \alpha_2/R$. In [24], the authors have shown that the synchronization time is proportional to $1/\lambda_2$, thus we obtained the minimal synchronization time $t_{\text{sync,min}} \propto 1/\lambda_2 \propto R$. We present the simulation results of $t_{\text{sync,min}}$ in Fig. 3(b), and the shape of the curve is similar to that of R in Fig. 2(b). The results we have obtained so far indicate that the generalized adaptive coupling method not only can obtain the best synchronizability by minimizing the eigenratio R, but also allows the network to achieve the fastest synchronization at the same α value by choosing γ close to γ_c, where the synchronization time is minimal for all the combinations of α and γ.

It has been confirmed that the original adaptive coupling method will make the network synchronizability better than the symmetric coupling method [14], here we give a comparison between the best case of generalized adaptive coupling method ($\alpha \approx 0.92$) and the original one ($\alpha = 0$). From Figs. 2(b) and 3(b) it is obtained that the eigenratio decreases from about 7.0 to 3.5, which means that for the given scale-free network of $N=1000$ nodes the range of coupling strength where synchronization can be achieved is doubled. Further more, the synchronization time decreases from about 90.3 to 30.4. Moreover, at the optimal point, R does not increase with the system size N, while R increases with N when deviates from the optimal point. According to our previous analysis of weighted networks [19], R is mainly determined by the maximal and minimal intensities as $R \approx S_{\max}/S_{\min} \approx [k_{\max}/k_{\min}]^{1-\theta} = [k_{\max}/k_{\min}]^{(1-\alpha)/2}$, where $k_{\max}$ increases with N in scale-free network. In this sense, the enhancement of synchronization by the generalized adaptive scheme can be very significant in large networks when taking $\alpha > 0$, i.e., when the capacity of the nodes in the network is limited. It is also interesting to compare the adaptive network to static weighted network with $w_{ij} = 1/k_i^{\alpha}$ [8]. In the static networks, we have $R \approx S_{\max}/S_{\min} \approx [k_{\max}/k_{\min}]^{1-\alpha}$, which is different from adaptive networks, but in both cases the optimal synchronizability happens at $\alpha = 1$.

We also investigate the synchronization property of GANs with half-bounded synchronization region and no essential difference has been found.

In conclusion, to study the interaction of network structure and dynamical property, we generalize the adaptive coupling method by regulating the input coupling strength by each node's degree, which takes into account not only the local dynamical information but also the local structure information $1/k_i^{\alpha}$, which represents the relative capacity of a node as a function of the degree. We have shown both numerically and analytically that the input coupling strength of the network after adaptation displays a power-law dependence on the degree, and the exponent θ is controlled by α according to Eq. (22). The results provide us a deeper understanding how the local interplay between structure and dynamics can lead to profound changes in the organization of large-scale structure and collective dynamics in complex networks. Interestingly, limitation in the node capacity (the region $\alpha > 0$) is found to enhance synchronization. This result has meaningful implications in understanding and manipulation of consensus in social dynamics.

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[1] D. J. Watts and S. H. Strogatz, Nature (London) **393**, 440 (1998).
[2] A.-L. Barabási and R. Albert, Science **286**, 509 (1999).
[3] R. Albert and A.-L. Barabási, Rev. Mod. Phys. **74**, 47 (2002).
[4] S. N. Dorogovtsev and J. F. F. Mendes, Adv. Phys. **51**, 1079 (2002).
[5] M. E. J. Newman, SIAM Rev. **45**, 167 (2003).
[6] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, Phys. Rep. **424**, 175 (2006).
[7] A. Arenas, A. Diaz-Guilera, J. Kurths, Y. Moreno, and C. S. Zhou, Phys. Rep. **469**, 93 (2008).
[8] A. E. Motter, C. Zhou, and J. Kurths, Phys. Rev. E **71**, 016116 (2005).
[9] G. Yan, T. Zhou, J. Wang, Z.-Q. Fu, and B.-H. Wang, Chin. Phys. Lett. **22**, 510 (2005).
[10] Z. Wu, L. A. Braunstein, S. Havlin, and H. E. Stanley, Phys. Rev. Lett. **96**, 148702 (2006).
[11] J.-P. Onnela, J. Saramäki, J. Hyvönen, G. Szabó, D. Lazer, K. Kaski, J. Kertész, and A.-L. Barabási, Proc. Natl. Acad. Sci. U.S.A. **104**, 7332 (2007).
[12] T. Gross, Carlos J. Dommar D'Lima, and B. Blasius, Phys. Rev. Lett. **96**, 208701 (2006).
[13] J. Ito and K. Kaneko, Phys. Rev. Lett. **88**, 028701 (2001); Phys. Rev. E **67**, 046226 (2003).
[14] C. Zhou and J. Kurths, Phys. Rev. Lett. **96**, 164102 (2006).
[15] Y. Wu, Y. Shang, M. Chen, C. Zhou, and J. Kurths, Chaos **18**, 037111 (2008).
[16] M. Chen, Y. Shang, C. Zhou, Y. Wu, and J. Kurths, Chaos **19**, 013105 (2009).
[17] L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. **80**, 2109 (1998).
[18] M. Barahona and L. M. Pecora, Phys. Rev. Lett. **89**, 054101 (2002).
[19] C. Zhou, A. E. Motter, and J. Kurths, Phys. Rev. Lett. **96**, 034101 (2006).
[20] D.-U. Hwang, M. Chavez, A. Amann, and S. Boccaletti, Phys. Rev. Lett. **94**, 138701 (2005).
[21] M. Chavez, D.-U. Hwang, A. Amann, H. G. E. Hentschel, and S. Boccaletti, Phys. Rev. Lett. **94**, 218701 (2005).
[22] M. Zhao, T. Zhou, B.-H. Wang, Q. Ou, and J. Ren, Eur. Phys. J. B **53**, 375 (2006).
[23] T. Nishikawa, A. E. Motter, Y.-C. Lai, and F. C. Hoppensteadt, Phys. Rev. Lett. **91**, 014101 (2003).
[24] J. A. Almendral and A. Díaz-Guilera, New J. Phys. **9**, 187 (2007).